A POSITIVE MODEL OF EXPENDITURE GROWTH

Michael Dothan, Mike Hand, Kawika Pierson and Fred Thompson
Atkinson Graduate School of Management, Willamette University, 900 State Street, Salem OR 97301 USA

ABSTRACT

A good positive model is merely a good normative model run backwards. Here, we test a positive model of the budget process, where the main goal of jurisdictions, which face a hard budget constraint, is stabilizing expenditure growth, taking existing revenue structures and fiscal assets (savings) as given, and treating revenue and savings growth as continuous-time, continuous-state stochastic processes. Our empirical tests address two reduced-form specifications derived from our positive model. The first predicts future spending as a function of a set of current (in the period of budget formulation) state variables, including a jurisdiction’s savings. The second predicts a jurisdiction’s savings as a linear-additive function of past spending, the rate of spending growth, and revenue volatility. We estimated both specifications using OLS and 1995-2006 census data for >14,000 municipalities. The first specification produces results that are consistent with our expectations, but not perfectly so: spending growth is more stable than revenue growth as the model predicts, but less stable than we anticipated. Moreover, spending is less responsive to savings than we expected. Consequently, our reduced-form specification barely outperforms a simple exponential smoothing model. Perhaps, this is the case because we used a “representative” municipality to calculate this result. Or, because budgetary decision makers are more myopic than we hypothesized. The results for the second specification are perfectly consistent with our expectations. This is arguably a fairly strong outcome: savings levels are uniquely determined in this specification and many observers will find the predicted result counterintuitive.

Keywords: Process • Mechanism • Martingale methods • Optimal stopping models

JEL Classification Numbers: H71 • H72

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2 Corresponding author (fthompson@willamette.edu).
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1 Introduction

In the standard economic model, governments are supposed to maximize the utility of the citizenry or a representative individual thereof (e.g., the median voter). Then, if governments do what they should do, the median voter’s demand for publicly provided goods and services, together with the supply of goods and services, will determine public spending, so that spending for a given service is a function of the median voter’s income, the tax price of the service, and the size of the jurisdiction. This model works exceptionally well for some purposes (why high-income school districts spend more than low-income districts) and fairly well for many others (making sense of public spending after the fact). However, it is an exceedingly blunt instrument for explaining “how it is that public funds are spent for one thing rather than another” (Key 1940: 1137). Absent a significant inertial component, it is practically useless for predicting public spending from one year to the next.

The weaknesses of the standard model can be attributed to various factors. Crecine, for example, focuses on its inattention to the citizenry’s agents, “the organizational decision-makers and problem solvers who structure complex problems, generate alternatives, and make choices” (Crecine 1969: 20). We focus on their core problem, maximizing the expected utility of the citizenry, a function of the mean and variance of future public and private consumption, in the face of uncertainty about the future. We emphasize the variance of public and private consumption because we assume that citizens are risk averse with respect to consumption and relatively more risk averse with respect to public than to private consumption.

2 The Model

To show that the budget actor’s problem can be formulated with some precision and to show how a computational solution to that problem can be found, we make two analytic turns. First, we specify the aspirational purpose served by the budget process. Second, because the problem concerns an
irreversible process that takes place over time, in which the order in which actions and events take place affects their outcomes, we embrace a set of powerful, new mathematical tools – optimal control and martingale theory – that allow us to find precise computational solutions to problems involving dynamic real processes.

The budget actor’s problem consists of making global spending decisions to maximize the expected utility of the citizenry, a function of the mean and variance of future public and private consumption, subject to a present-value constraint and considerable uncertainty about future cash inflows. A hard budget constraint means that these enterprises can neither force central banks to buy their bonds nor expect a higher jurisdiction or authority to rescue them from fiscal distress (Rodden, Eskeland, and Litvack, 2003). A hard budget constraint provides budget actors with an intentional problem: stabilizing service deliveries in the face of revenue uncertainties, largely a function of the business cycle; an opportunity: to save for a ‘rainy day;’ and a constraint: the present value of future revenues plus net savings (financial assets minus financial liabilities) must be equal to or greater than the present value of future spending (Buiter, 1990: 63-7). In financial economics, it is easy to show there is an optimal solution to the present-value spending problem, which is the key to budget actor’s global spending problem, given a hard budget constraint.

Actually solving the budget actor’s global spending problem in real time, especially from one period to the next, is much harder computationally, even where the problem can be formulated in a straightforward manner, as we think it can be here.

2.1 A Proposed Mechanism

Mechanisms are the constructs we use to explain why processes turn out the way they do. For

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1 More precisely, Buiter expresses the solvency constraint of a jurisdiction subject to the ‘no-Ponzi game’ condition as: PV of exhaustive spending ≤ {(assets – debts) + PV (taxes – transfers)}, where PV denotes the present value of a cash flow.
example, evolution is a process; natural selection is one of the mechanisms that determine its outcome. Similarly, budgeting is a process and balancing and precedent are among the mechanisms that determine its outcome. Real processes take place over time. Consequently, where real processes are concerned, mechanism-based explanations look like algorithms or, perhaps, scenarios, when the analytical tradeoff tilts away from abstraction toward veridical realism.

The question is how best to deal with the uncertainty facing the budget actor? Like Padgett (1981) and George A. Krause (2003), we choose to formalize conditions, which for reasons of purpose and method Crecine and other budget theorists of his school leave tacit and open-ended. The way we do this is to treat the budget actor’s problem as a particular kind of closed-form, stochastic-choice problem called an optimal stopping problem. Stopping problems can be laid out two ways: deciding how long to wait before choosing (e.g., Carpenter 2010) or making the best stochastic choice possible at a specified time. In this instance, both interpretations do some injury to reality. But the latter, setting a global spending level according to a predetermined schedule, seems to fit the stylized facts of the budget formulation process somewhat better than the alternative of deciding how long to delay before setting a global spending level. Budget actors are usually described as making global spending decisions in the period of budget formulation and then sequentially repeating this process in each subsequent period of execution (when the spending occurs), basing future spending levels on the expected outcomes of their choices and current state variables in the period in which the budget is formulated (Penner and Steuerle, 2004). Hence, we are saying that the budget actors’ problem is to determine an endogenous optimal expenditure process $x_t$, given a budget savings or reserve account, $H_t$, subject to the binding constraint that the risk-adjusted present value of future spending must be equal to or less than net financial savings, $H_o$.

\footnote{In jurisdictions where re-budgeting is the norm, the optimal stopping problem should, perhaps, be expressed in terms of delay (Caiden and Wildavsky 1974; Wildavsky 1975).}
plus the risk adjusted present value of future revenues.

Finally, one of the characteristics of deterministic optimal stopping problems is that actors’ preferences, especially their risk preferences, drive results. Consequently, “criteria must be advanced that can enable scholars to empirically demarcate between risk-aversion, risk-neutrality, and risk-seeking behaviors” (Krause 2003: 171). Here, we assume that budget actors have a utility function over the spending stream discounted by the combined value of population growth and inflation and we denote this combined growth rate by $\alpha$, constant relative risk aversion $\gamma$ and rate of time preference $\phi$. Other relevant state variables are the market price of risk, which we denote by $\theta$, the risk-adjusted capitalization rate of revenue, denoted by $k$, and the interest rate paid on savings (assumed to be invested in bonds), denoted by $r$.

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3 Arrow-Pratt risk aversion formalizes the everyday notion of prudence, given a concave von Neumann-Morgenstern utility function. The specific measure we use is called constant relative risk aversion because, in this case, it expresses the weight given to shortfalls (spending below some target level) relative to windfalls (spending above the target) as a constant ratio (based on the utility function's second derivative). As such, it measures willingness to sacrifice current and future spending (by holding fund balances) to minimize spending variability around a target. Here, we make the explicit assumption that the relative risk aversion for public consumption is a multiple of the relative risk aversion of private consumption, estimated by Chetty (2006) to be greater than 1 but probably less than 2. We assume that relative risk aversion for public consumption is greater (see Barsky, Juster, Kimball & Shapiro 1997).

4 The rate of time preference formalizes the everyday notion of inter-temporal bias. It measures people's willingness to reduce future spending to increase current spending.

5 We assume that, so long as a jurisdiction avoids Ponzi finance, it can lend or borrow at the same rate.
The assumption that a government’s existing revenue structures and volatilities are givens can be formalized as meaning that the entity’s revenue stream $y_t$ is exogenous to the model and its dynamics, over very short time intervals, $dt$ are the sum of constant growth at rate $\mu$ and a random shock represented by an increment of Brownian motion $B_t$, a continuous-time symmetric random walk whose increments have normal distribution with zero mean and variance $\tau$, scaled by a volatility parameter $\sigma$.

\[
\frac{dy_t}{y_t} = \mu dt + \sigma dB_t
\]  

(1)

The specification of revenue dynamics in Equation (1) implies the following revenue stream:

\[
y_t = y_0 \exp \left( \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma B_t \right)
\]  

(2)

The random revenue process in Equation (2) is called exponential Brownian motion and is characterized by an infinite number of potential sample paths with intermittent periods of exponential growth and decay. At any future time, the distribution of exponential Brownian motion is log-normal.

2.2 Optimal Spending

To find the solution to the budget actors’ problem, we exploit an analogy between government expenditures and investor’s consumption, and between government revenues and investor’s labor income. Merton (1971) solved an optimal consumption and portfolio problem for an investor with no labor income, using continuous-time dynamic programming. We modify Merton’s problem to include random government revenue whose dynamics are represented by an exponential Brownian motion. We assume that financial markets are rich enough to allow hedging the uncertainty in future government revenue, but that legal and institutional factors restrict the government to implementing the optimal expenditure policy through investing the reserve account in risk-free bonds only or, if
the reserve account is negative, borrowing at the risk free rate. With that assumption, the solution of
the actor’s problem is given in Equations (3) and (4).

\[ x_t = \nu \left( H_t + \frac{y_t}{k-\mu} \right) \]  

(3)

\[ H_t = \exp \left[ (r-\nu)t \right] H_0 + \left( 1 - \frac{\nu}{k-\mu} \right) \int_0^t \exp \left[ (r-\nu)(t-s) \right] y_s ds \]  

(4)

Equation (3) says that optimal expenditure is a constant fraction \( \nu \) of wealth, defined as the value of
the reserve account (which could be negative as well as positive) plus the risk-adjusted present
value of future revenues. The fraction \( \nu \) represents the optimal spending rate from wealth, and is
given by:

\[ \nu = \frac{(r-\alpha)(\gamma-1) + \phi}{\gamma} + \frac{(\gamma-1)\theta^2}{2\gamma^2} \]  

(5)

Table 1 presents the comparative statics of optimal spending from wealth. We assume a
representative or base case with standard parameter values for \( r, \theta, \) and \( k \) of 5\%, .45\%, and 9.5\%,
for \( \mu, \sigma, \) and \( \alpha \) of 6\%, 10\%, and 4\%, and the assumed values of \( \phi \) and \( \gamma \) of 0\% (no inter-temporal
myopia) and 5.0. Table 1 shows that in the base case optimal spending is 2.42 percent of wealth.
Increasing \( r, \phi, \) and \( \theta \) from base-case values increases spending. Increasing \( \alpha \) and \( \gamma \) from base-case
values reduces spending. Other things equal, changes in expected long-run growth of revenue \( \mu \)
would have no effect on spending rates (\( \nu \) expressed as a proportion of wealth). However, because
an increase in \( \mu \) increases wealth, increased revenue growth would also allow higher spending
levels. Similarly, increases in the volatility of revenue \( \sigma \) would reduce wealth by increasing the
market price of risk \( \theta \).
Equation (4) describes the optimal value of the reserve account as a function of the history of revenue $y_s$ for $0 \leq s \leq t$. Because our analysis is in continuous time, our formula reflects continuous compounding of the reserve account at the annual rate $r$. The formula follows by integration from the compounding equation, where $x_t$ is given by Equation (3). Recursive calculation using Equation (6) represents a very good approximation to Equation (4) when $\Delta t$ is one day or less.

$$H_t = (1 + r \Delta t) H_{t-\Delta t} + y_t \Delta t - x_t \Delta t$$ (6)

### 2.3 Optimal Reserve Account

To investigate the optimal size of the reserve account $H_t$, it is convenient to consider the optimal reserve to spending ratio $H_t / y_t$. From Equation (4) we can compute this expected ratio when time goes to infinity. Table 1 also shows the comparative statics of this ratio. In the base case, the optimal long-run expected reserve account is 93.57 percent of annual spending (11 months). Compared with the base case, increasing bond rate $r$, revenue volatility $\sigma$, combined growth rate $\alpha$, budget actor's risk aversion $\gamma$, and market price of risk $\theta$ increase the size of the long-run optimal reserve account. Increasing expected long-run growth of revenue $\mu$ reduces the size of the optimal reserve account and may even convert it into a debt account, as in row 3 of Table 1.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Optimal Expenditure as Percentage of Wealth</th>
<th>Long-Run Optimal Reserve as Percentage of Revenue</th>
<th>Long-Run Optimal Surplus as Percentage of Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>2.42%</td>
<td>93.57%</td>
<td>0.94%</td>
</tr>
<tr>
<td>$r = 5.25%$</td>
<td>2.53%</td>
<td>108.84%</td>
<td>0.82%</td>
</tr>
<tr>
<td>$\mu = 7.25%$</td>
<td>2.42%</td>
<td>-205.87%</td>
<td>-2.57%</td>
</tr>
<tr>
<td>$\sigma = 0.25%$</td>
<td>2.42%</td>
<td>218.89%</td>
<td>2.08%</td>
</tr>
<tr>
<td>$\alpha = 4.25%$</td>
<td>2.22%</td>
<td>347.83%</td>
<td>3.48%</td>
</tr>
<tr>
<td>$\phi = 0.25%$</td>
<td>2.47%</td>
<td>34.58%</td>
<td>0.35%</td>
</tr>
<tr>
<td>$\gamma = 0.00%$</td>
<td>2.24%</td>
<td>321.54%</td>
<td>3.22%</td>
</tr>
<tr>
<td>$\theta = 0.50$</td>
<td>2.80%</td>
<td>273.08%</td>
<td>2.74%</td>
</tr>
</tbody>
</table>
Similarly, although less happily, changing actors’ time preference $\phi$ in our model so that they prefer current spending to future spending, also reduces the size of the reserve account, as in row 6 of Table 1, and, if the bias reflected in $\phi$ is large enough, could convert it to a debt account. Of course, a good positive model of the budget process should explain avoidable financial distress and not merely desirable outcomes (Holcombe and Sobel 1997; Hou 2006).

2.4 Spending from Revenue and the Optimal Surplus

Equations (3) and (7) imply that the optimal long-run ratio of spending to revenue is as shown in Equation (8). In turn, Equation (8) can be rearranged to show the long-run optimal ratio of spending to the long-run optimal ratio of surplus to revenue, Equation (9).

\[
\frac{\nu}{k - \mu - \mu - r + \nu - \sigma^2}
\]

3 Empirical Assessment

Hedström reminds us that while statistical analysis is not explanation, it is “important for testing proposed explanations” (2005 23). So, how well does our explanation work? The answer to that question depends, of course, upon the specification tested and the analytical alternatives. In this case, we have a fully identified model. Consequently, it would be possible, given sufficient resources, to predict spending levels and reserves for any jurisdiction and time period with considerable precision, using normative standards for risk aversion and time preferences, to generate an appropriate counterfactual, expressed in terms of spending or saving or both, and thereby test whether actual spending/saving behavior differs significantly from the counterfactual.

Alas, we lack the data on all the state variables reflected in the model and, thus, needed to carry out this analysis. As a result we have adopted a less satisfactory expedient, we tested two reduced-
form specifications derived from our positive model. The first predicts future spending as a function of a reduced set of current (in the period of budget formulation) state variables, including a jurisdiction’s reserves. The second predicts a jurisdiction’s savings as a linear-additive function of past spending, the rate of spending growth, and revenue volatility. We estimated both specifications using OLS and 1993-2006 census-of-governments data for about 20,000 municipalities.

3.1 Empirical Models

Our first reduced-form specification reflects the assumption that spending \((x)\) depends upon revenue growth \((\mu)\) and savings \((H)\). Combining Equations (3) and (5), we obtain:

\[
x_t = \left[ \frac{(r - \alpha)(y - 1) + \phi (y - 1)\theta^2}{\gamma} + \frac{(y - 1)\theta^2}{2\gamma^2} \right] \cdot \left( H_t + \frac{y_t}{k - \mu} \right)
\]  

Then, using a first-order Taylor series approximation of the non-linear function for \(x_t\), its partial derivatives with respect to \((r-\alpha), H_t,\) and \(y_t\) are:

\[
\frac{\partial x_t}{\partial (r - \alpha)} = \left[ \frac{(y - 1)}{\gamma} \right] \cdot \left( H_t + \frac{y_t}{k - \mu} \right) 
\]  

\[
\frac{\partial x_t}{\partial (H_t)} = \left[ \frac{(r - \alpha)(y - 1) + \phi (y - 1)\theta^2}{\gamma} + \frac{(y - 1)\theta^2}{2\gamma^2} \right] 
\]  

\[
\frac{\partial x_t}{\partial (y_t)} = \frac{1}{k - \mu} \cdot \left[ \frac{(r - \alpha)(y - 1) + \phi (y - 1)\theta^2}{\gamma} + \frac{(y - 1)\theta^2}{2\gamma^2} \right]
\]  

Consequently, we can model spending as:

\[
\Delta x_t = c + \beta_1 \Delta (r - \alpha) + \beta_2 \Delta H_t + \beta_3 \Delta y_t + \epsilon
\]  

where \(t = 1\) is the period in which the budget is executed, \(t = 0\) is the period in which the budget is formulated, and \(t = -1\) is the period immediately prior to the period of formulation. Each regression coefficient is an estimate of one of the partial derivatives in equations 11-13, leaving us with three equations in three unknowns allowing us to convert these coefficients into estimates of
Our second reduced-form specification goes to the assumption that reserves \((H)\) depend upon revenue volatility \((\sigma)\) and revenue growth \((\mu)\). Given Equation (6) and assuming current state variables are held constant, the model implies:

\[
\frac{H}{y} = c + \beta_1 \sigma + \beta_2 \mu + \epsilon
\]

(15)

Of course, this prediction follows not only from Equation (6) but also the standard advice offered by specialists in public financial management. The Government Finance Officers Association recommends, for example, that municipalities maintain reserves equal to at least 16 percent of revenues, plus additions to cope with the following hazards: revenue volatility, infrastructure failures and vulnerability to extreme events such as fire, flood, tornados, or earthquakes.

3.2 Data

Data are from the Census Bureau's Annual Survey of Governments for all cities, towns, boroughs, and villages in the database (Government Type = 2), covering nearly 20 thousand municipalities. The sample period covers the years 1992 and 1995 – 2006, all the years for which Individual Unit Files are currently available, and provides over 80 thousand data-years of observations. However, all municipalities report only once per lustrum (the 2\(^{nd}\) and 7\(^{th}\) year of each decade). A random sample of municipalities are surveyed in the intervening years. Consequently over half all the theoretically possible data years are missing. Because we needed three consecutive

\footnote{We used municipal data for three reasons: municipalities are fiscally autonomous jurisdictions, which are subject to a hard budget constraint (Lowry 2001); there are lots of them, which means that we have a very large sample to work with; they are subject to mechanisms that constrain agents to attend to the preferences of the citizenry (Wittman, 1989; Wintrobe, 1987; Tiebout, 1956; Breton, 1977 & 1996; Fischel, 2005).}
years of data to perform our first test and at least five years of data overall to perform our second, we performed the first on less than 15 thousand observations and the second on 8,000.

The financial measures we used were extracted from Government Finance Statistics, Individual Unit Files and matched against Government Integrated Directories for population and other entity attributes. The Individual Units data contain details for all “income statement” accounts, including revenues, expenditures, and transfers. The Census files also provide “balance sheet” data such as cash and debt.

To estimate our empirical models, we have data on spending \( x \), revenue \( y \), revenue growth \( \mu \), reserves \( H \), and revenue volatility \( \sigma \). REVENUE and SPENDING are taken directly from the census reports.

GROWTH is the change in population from year \( t-n \) to \( t \) multiplied by the change in the CPI.

VOLATILITY is a metric for revenue volatility.

To calculate GROWTH and VOLATILITY, we decomposed the historical data for each jurisdiction into two components: a trend and a random component. The trend is simply the mean of past growth rates (where revenue is concerned, \( \mu \) in our model). The random component is the variance. The variance is the average of the squared differences between actual growth rates and the mean growth rate – or mean squared error. The standard deviation of the distribution is simply the square root of the variance (where revenue is concerned, \( \sigma \) in our model).

RESERVES/REVENUE is a proxy for reserves \( H \) in our model. RESERVES/REVENUE are computed as total fund balances (or fund equity) at the end of each fiscal year, deflated by revenue per month. This represents the number of months a government can operate without collecting additional revenue. Because of the dramatic disparity in the reserves to revenue ratios, ranging in value from 0 to 6336 with coefficient of skewness = 67.6, the natural log of this ratio was employed as the response to the second model tested.

Additional variables used to estimate our empirical models include:
DEBT PER CAPITA is total debt/total population (inverse proxy for revenue volatility);
REVDIV is a revenue diversification index, or \([1 + r_1/R)(1 + r_2/R)...(1 + r_n/R) - 1]\), where \(r\) is revenue from a specific source and \(R\) is total revenue (proxy for revenue uncertainty);
SIZE is the log of total population;
STATE REVENUE is the total state revenue received/total revenue (inverse proxy for revenue uncertainty);
FYE is a series of dummy variables indicating the quarter of the fiscal year end;
STATE is a dummy variable indicating the state; and
YEAR is a dummy variable indicating the year.

Summary statistics for model variables are displayed in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves/Revenue</td>
<td>80,150</td>
<td>13.08</td>
<td>8.00</td>
<td>44.6</td>
</tr>
<tr>
<td>ln(Reserves/Revenue)</td>
<td>75,121</td>
<td>2.09</td>
<td>2.15</td>
<td>1.01</td>
</tr>
<tr>
<td>Volatility</td>
<td>16,634</td>
<td>0.14</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>Debt Per Capita</td>
<td>80,150</td>
<td>2.68</td>
<td>0.68</td>
<td>33.52</td>
</tr>
<tr>
<td>Limited Revenue</td>
<td>77,870</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Population</td>
<td>81,023</td>
<td>20,863</td>
<td>2.123</td>
<td>133,149</td>
</tr>
<tr>
<td>ln(Population)</td>
<td>81,023</td>
<td>7.83</td>
<td>7.66</td>
<td>2.06</td>
</tr>
<tr>
<td>Growth</td>
<td>41,979</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>State Revenue</td>
<td>80,335</td>
<td>0.17</td>
<td>0.12</td>
<td>0.16</td>
</tr>
</tbody>
</table>

3.3 Test 1

We used the following OLS model to estimate the determinants of expected municipal spending.

\[
\Delta \text{SPENDING GROWTH}_{1,0} = c + \beta_1 (r - \text{GROWTH}) + \beta_2 (\Delta \text{RESERVES}_{0.1}) + \beta_3 (\Delta \text{REVENUE}_{0.1}) + \varepsilon
\]

The results shown in Table 3 indicate that municipal spending is, in fact, more stable than revenue.
and responsive to changes in reserves levels. Moreover, the model works quite well. It outperforms all of the naïve formulations we tested, including the gold standard in budget forecasting (Downs and Rocke, 1983, 1984), an exponential smoothing model where revenue growth predicts spending growth (.54 > .48).

The problem arises when we look at specific numerical results given our expectations. Because each regression slope is an estimate of one of the partial derivatives in equations 11-13, giving us three equations in three unknowns, it is a simple matter to convert our slope estimates into values for $\gamma$, $\phi$, and $\kappa - \mu$. Using plausible values for our population means, we obtain a range of values for relative risk aversion ($\gamma$) of 1 to 6.6 and of temporal bias ($\phi$) of .18 to 1.05. Both of these ranges are reasonably consistent with our expectations. However, the implicit discount rate implied by these results is nearly 50% percent ($\kappa - \mu = .391$). What this means is that reserves materially (albeit not statistically) weigh much less in the budget process than is optimal and recent revenue changes, far more. In other words, the spending outcomes that result from the municipal budget process are far more short-sighted or myopic than they should be, if budget decision makers were, in fact, trying to optimally stabilize revenue growth over time.

### 3.4 Test 2

We used the following OLS model to estimate the determinants of expected municipal reserves (fund balances):

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-Stat</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.14</td>
<td>0.30</td>
<td>0.46</td>
<td>NS</td>
</tr>
<tr>
<td>Discount Rate - Growth</td>
<td>-0.42</td>
<td>0.80</td>
<td>-1.10</td>
<td>.06*</td>
</tr>
<tr>
<td>Change in Reserves</td>
<td>0.18</td>
<td>0.02</td>
<td>9.52</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>Change in Revenue</td>
<td>0.46</td>
<td>0.14</td>
<td>3.26</td>
<td>&lt;.001***</td>
</tr>
</tbody>
</table>

**Adjusted R²: 0.54**

* *, **, *** indicate significance at p < .10, .05, and .01, based on two-tailed tests.

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Comment [REV1]: It is not entirely correct to describe exponential smoothing (ES) as a naïve model in this case. ES is closely related to state-space models of unobserved components. In particular, ES is the MSE-optimal filter when the data-generating process is a latent random walk signal buried in white noise measurement error (e.g., a regime switching model, where the time of the switch is unknown). The optimal smoothing parameter, moreover, depends only on the signal / noise ratio (that is, the random walk error variance relative to the measurement error variance), in which case the signal / noise ratio and hence the optimal smoothing parameter are time-varying. The particle filter facilitates both optimal parameter estimation and optimal tracking of the time-varying volatility, making for real-time ES with an optimally time-varying smoothing parameter.
\[ \ln(\text{SAVINGS/REVENUE}_i) = \alpha + \beta_1 \text{CV REVENUE}_i + \beta_2 \text{GROWTH}_i + \beta_3 \text{DEBT PER-CAPITA}_i + \beta_4 \text{LIMITED REVENUE}_i + \beta_5 \text{SIZE}_i + \beta_6 \text{STATE REVENUE}_i + \sum \beta_k \text{FYE}_k + \sum \beta_m \text{STATE}_m + \sum \beta_t \text{YEAR}_t \]

where \( i \) represents the \( i \)th government entity and \( t \) represents the year of observation. This model is driven by \( \text{CV REVENUE} \) and \( \text{GROWTH} \). However, the model further implies that more debt, a narrower tax base, and a larger population should reduce savings. The latter conclusion follows from inventory models, which hold that the size of inventory required to buffer against uncertainty increases with the square-root of operational scale (i.e., less than proportionally). We further assumed that greater reliance on state aid would reduce the effects of own-source revenue instability and, therefore, reduce municipal saving.

### TABLE 4: Determinants of Saving

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-Stat</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.14</td>
<td>0.30</td>
<td>3.77</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.23</td>
<td>0.07</td>
<td>3.20</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>Growth</td>
<td>1.95</td>
<td>0.19</td>
<td>10.43</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>Debt Per Capita</td>
<td>0.00</td>
<td>0.00</td>
<td>-2.00</td>
<td>.06*</td>
</tr>
<tr>
<td>Revenue Diversification</td>
<td>12.20</td>
<td>3.76</td>
<td>3.25</td>
<td>.001***</td>
</tr>
<tr>
<td>( \ln(\text{Population}) )</td>
<td>-0.02</td>
<td>0.01</td>
<td>-3.11</td>
<td>.002***</td>
</tr>
<tr>
<td>State Aid</td>
<td>-0.30</td>
<td>0.09</td>
<td>-3.42</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>FYE Quarter Dummy</td>
<td>***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year Dummy</td>
<td>***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>State Dummy</td>
<td>***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For brevity, the quarter-specific, year-specific, and state-specific coefficients are not reported.

Fiscal year-end, state, and year indicators are included in the regressions, but are not reported in Table 4. All regression specifications report t-statistics and p-values. The analysis shows that municipalities with more volatile revenues, more limited revenue sources, and with higher growth have higher savings, while larger municipalities, those with greater debt, and more state funding...
save less. All of the model's explanatory variables are significant, and in the predicted direction.

The explanatory power of the model is represented by an $R^2$ of 23.8 percent, which is similar to that reported in prior literature (e.g., Gore 2009\(^7\)).

As in our first test, we can use the estimated coefficients plus the mean values of the determinants, plus the assumed values for constant relative risk aversion $\gamma$ of 5.0 and rate of time preference $\phi$ of 0 (see above), to calculate a savings level for a “representative municipality” (as in Table 1). Overall, the average municipality was predicted to carry 11 months of expenditures in savings. In fact, it carried a mean of 13 months. We interpret these results as consistent with expectations. Indeed, this is arguably a fairly strong outcome: savings levels are uniquely determined in this specification and many observers will find the predicted outcome counterintuitive and the actual outcome incredible (for example, when we submitted a normative version of this model for review, one reviewer expressed his disbelief that as a practical matter jurisdictions would ever hold anything like the savings called for by the model).

3.5 Findings

With regard to its comparative static predictions of spending and reserves, the model does fairly well overall. The first specification produces results that are consistent with our expectations, but breaks down somewhat when we look at specific numerical results: spending growth is more stable than revenue growth as the model predicts, but is more responsive to revenue changes than we predicted. Perhaps, this is due to misspecification. But as of now it looks as if budgetary decision makers are simply more myopic than we assumed. In contrast, the results for the second specification are perfectly consistent with our expectations. This is arguably a fairly strong outcome: reserve levels are uniquely determined in this specification and many observers will find

\(^7\)In point of fact, the empirical model estimated here and its results are close enough to Gore’s to be viewed as replicating her findings, albeit not her inferences.
the predicted result counterintuitive.

Arguably, the systematic deviations from our numerical expectations raise interesting questions about the motivational forces underlying the behavior of budget makers, insofar as they tend to emphasize the apparent myopia of budget process and downplay temporal bias and risk aversion on the part of budget makers. At present, this looks to be worth further investigation.

The next step in our research program lies testing the spending model using the next five years of municipal census data. These data are now available but in an unedited form.

4 Conclusions and Discussion

We have tested a model, which presumes that budgeting is a purposeful enterprise, aimed at an identifiable normative optimum. This is how we usually study business, for example. We start by assuming that businesses maximize net-present-value benefits to their stakeholders. After about a century of debate, we have determined that this usually means maximizing the present value of future free-cash flows thrown off by inventory turnover, which resolves the recognition question (when to count performance), the standing question (whose benefits count), and the question of idiosyncratic risk. We justify this norm by referencing Fisher’s separation theorem, which argues that shareholders should care only about a business’s profitability and not the means by which it is realized and the claim that shareholders ought to be indifferent to enterprise-specific or

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8This norm doesn’t apply to all businesses, banks, for example, create value by holding financial inventories; using equity, risk pooling, and hedges to manage inventory risk. It is not unreasonable to suggest that many of them got into trouble precisely because they lost sight of this purpose and instead got caught up in maximizing yields.

9This assumption isn’t necessarily valid. Shareholders often appear to have preferences for things other than the properties of a business’s returns. Social responsibility may be an example. A preference for profitable growth stocks is probably a better one. That some investors get utility from
idiosyncratic risk, because they can diversify it away. Then, once we have established a satisfactory norm that can be used to discern appropriate action, we can, where there is reason to do so, investigate the consequences of conditions that are intrinsic to the situation at hand: asymmetric information, bounded rationality, peculiar business strategies, path dependencies, or shifting internal coalitions.

Of course, that is precisely the analytic strategy we have pursued here. If you cannot say what success looks like you cannot say much about anything else of importance. The thing that is original about our analysis is the content of the normative optimum specified. Nevertheless, the logic of our formulation reflects a fairly commonplace observation: the goal of budget process is fiscal stability, which can in turn be derived from the proposition that the provision of stability is government’s primary role. Stability is something people want and often need. It is also one thing business cannot deliver; governments can. Broadly speaking, governments can create stable institutional frameworks that allow markets to work effectively, reduce the volatility of business cycles, thereby dampening systemic risk, by an array of risk-spreading, transfer programs aimed at mitigating idiosyncratic individual and collective hazards, and by underwrite the provision of various vital services – services which might otherwise often be unavailable precisely when most needed. Our only gloss on this nearly universal understanding of government’s purpose lies in using the formal language of optimization instead of the everyday language of safety, order, and security.

The assertion that stability is central to government purpose has very important implications for the study of public policy and administration. First, it distinguishes public from private enterprises. Private enterprises are primarily concerned with managing productivity. Governments are primarily concerned with managing risk. Second, a better understanding of government’s purpose can help being connected to a ‘winner,’ although they tend to be poorer investments than value stocks, is one possible explanation for the anomalous market premium they command.
scholars make interesting, surprising instrumental claims about its behavior. This is the case not only with respect to the budgetary process or machine bureaucracy, both of which are known to have stabilizing properties, but also to public personnel management (Bellante and Link, 1981; Buurman et al., 2012; Pfeifer, 2012) or agency behavior (Carpenter, 2010). Third, the distinction outlined here links public policies to politics. That management of risk and uncertainty is the basic function of government implies that risk preferences and attitudes are directly relevant to an understanding of partisan conflict. There is an extensive and growing literature in political science that takes precisely this position (see Thompson, Ellis, and Wildavsky, 1990; Ellis and Thompson, 1997; Thompson, Grendstad, and Selle, 1999; Swedlow, 2011a & 2011b). Putting stability front and center in the analysis of government behavior would allow these linkages to be explored and holds out to our field the potential for real intellectual progress.

Finally, on a narrower front, the model presented here provides a basis for advancing the “precise, abstract, realistic, and action-based explanations” (Hedström 2005: 1) of public spending, articulated in Crecine’s 1969 book Governmental Problem Solving (Crecine, 1969) and elegantly formalized by Padgett (1980, 1981), the so-called organizational-process models of budgetary behavior.

Unfortunately, their work has slipped into neglect. There are several reasons for neglecting

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10We say unfortunately because, in the period following the formulation of Crecine’s model, considerable progress was made toward a sophisticated understanding of the mechanisms and activities that influence the allocation of budgetary resources, which emphasizes the relative “controllability” of expenditure categories, transient preferences, and the stochastic effects of short-term expenditure solutions, see Downes and Rocke (1984), Bromily (1981), Bromily and Crecine (1980), Gist (1974) and Wanat (1974). Much of this literature is now overlooked, although thanks to the work of Kenneth Meier and Laurence O’Toole (2011) and Jonathan Bendor (2010) it has
organizational-process theories of budgeting, but their big flaw is that they are only half finished (Green and Thompson 2001). They explain spending at the program or agency level in the period in which a budget is executed in terms of the previous year’s spending for the same purpose and total spending growth. What this means that one must know how much revenue was deemed available to be spent overall in a period (or, in empirical analysis, was spent) before one can say how much spending would grow (or decrease) for a particular purpose. This is a remarkable accomplishment, of course, but it leaves us with theories that lack any real predictive power.11

In other words, the fundamental flaw of organizational-process budget models is that they treat total spending in the year of execution as exogenous to the model. Every version of the theory starts with the presumption that budgets must be balanced against a spending constraint and that this constraint is the principal driver of budgetary outcomes, but the formulation’s prime mover remains a deus ex machina. The mechanism – a continuous chain of intentional and causal links through which a global spending constraint is established – is missing.

What the organizational process approach to budgeting lacks and, therefore, what it needs is a mechanism that predicts global spending. That is what we provide here. Of course, as a practical matter, we doubt that budgetary decision makers use optimizing models that look like ours (save, garnered increased attention in recent years (see also Green and Thompson 2001; Jones 2002, 2003; Dezhbakhsh, Tohamy, and Aranson 2003; Tohamy, Dezhbakhsh, and Aranson 2006; Breunig and Koski 2006; Robinson, Caver, O’Toole, and Meier 2007).

11 In several versions of the model (e.g., Crecine 1969), the spending constraint is referred to as a revenue constraint, which gives it spurious proximity and materiality. Consequently, some students of the budgetary process have overlooked the incompleteness of the model (see, however, Davis, Dempster, and Wildavsky 1971, 1974; Downs and Rocke 1983, 1984; Choate and Thompson 1988).
perhaps, for the Chileans, see Frankel, 2011). Modeling the budget process as a set of differential equations is no more than an analytic convenience aimed at showing what spending behavior would look like if budget makers optimized performance.

Having found that observed spending/saving behavior is roughly consistent with the predictions of our model, the next step implied by the organizational-process approach is to determine the heuristics or decision rules budgetary decision makers actually use to produce these results. We are inclined to believe that our first order of business ought to be a clearer understanding of the determinants of budget myopia. Because the results that are called for by our model can be approximated by the geometric mean of revenue growth, doing so looks to be eminently feasible.
References


Downs, George W. and David M. Rocke. 1984. Theories of Budgetary Decision making and


